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STRESS STATE OF A CYLINDRICAL SHELL LOADED ALONG SEGMENTS OF THE DIRECTING CIRCLE

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Simple closed expressions for forces and moments in a circular cylindrical shell loaded uniformly by radial forces along segments of the directing circle are obtained by asymptotic synthesis methods (ASM) in [1, 2]. In practice, the pressure distribution can be significantly nonuniform, especially when a shell is in contact with other rigid bodies [3, 4]. For this reason, the results of [1, 2] are extended below to the case of a nonuniform distribution of external load. Unlike in [5], where a two-dimensional Fourier transform was employed, the solution in our case is constructed using an ASM. This method gives simple compact relations for forces and moments for a sufficiently general external load.

We denote the thickness and radius of the shell by h and R, and the elastic modulus and Poisson's ratio of the shell material by E and ν. We assume that the cross section $x = 0$, to which an external load is applied, is sufficiently distant from the shell edges, so that the influence of the edges on local bending can be ignored. That is, the shell is considered infinite in the axial direction. This simplifying assumption is acceptable for a self-balanced external loading which is cyclically symmetrical about the angular coordinate β composed of k normal forces applied periodically along the directing circle. Each of the forces P is distributed along a circular arc with length $2\beta_0 R$ by the law $f(\beta) = f(-\beta)$, so that

$$P = R\int\limits_{-\beta_0}^{\beta_0} f(\beta)\,d\beta = 2R\int\limits_{0}^{\beta_0} f(\beta)\,d\beta.$$

As in [1, 2], we represent the local stress state as the sum of the principal state and a simple boundary effect. We describe the former by the Schorer type equation [6]

$$\frac{\partial^4\Phi}{\partial\alpha^4} + c^2\frac{\partial^8\Phi}{\partial\beta^8} = \frac{R^2}{Eh}p(\alpha,\beta), \tag{1}$$

where $\alpha = x/R$, $c^2 = h^2/(12(1-\nu^2)R^2)$, $\Phi = \Phi(\alpha,\beta)$ is a resolving function, and $p(\alpha,\beta)$ is the density of the external radial load.

In the principal state, the bending moments $G_1^{\rm p}$ and $G_2^{\rm p}$ and the tangential forces $T_1^{\rm p}$ and $T_2^{\rm p}$ are expressed in terms of the function Φ as

$$G_1^{\rm p} = \nu G_2^{\rm p} = -\nu\frac{D}{R^2}\frac{\partial^6\Phi}{\partial\beta^6},\ D = \frac{Eh^3}{12(1-\nu^2)} = Ehc^2R^2,\ T_1^{\rm p} = -\frac{Eh}{R}\frac{\partial^4\Phi}{\partial\alpha^2\partial\beta^2},\ T_2^{\rm p} = 0. \tag{2}$$

The stress state of the simple boundary effect is given by the equation [1]

$$\frac{\partial^4 w}{\partial\alpha^4} + c^{-2}w = R^2D^{-1}p(\alpha,\beta),$$

which is written with respect to the lateral deflection (or radial displacement) $w = w(\alpha,\beta)$. This corresponds to the force factors

$$G_2^{\rm b} = \nu G_1^{\rm b} = -\nu\frac{D}{R^2}\frac{\partial^2 w}{\partial\alpha^2},\quad T_2^{\rm b} = -\frac{Eh}{R}w,\quad T_1^{\rm b} = 0.$$

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Note that here and below the superscripts p and b indicate that a factor belongs to the principal state or to the boundary effect, respectively.

The external-load density is given by $p(\alpha,\beta) = q(\beta)R^{-1}\delta(\alpha - 0)$. Here

$$q(\beta) = q\Big(\beta \pm \frac{2\pi}{k}\Big) = \begin{cases} f(\beta) & \text{for} \quad \beta \in [-\beta_0, \beta_0], \\ 0 & \text{for} \quad \beta \in \Big[-\frac{\pi}{k}, -\beta_0\Big) \bigcup \Big(\beta_0, \frac{\pi}{k}\Big]; \end{cases}$$

and $\delta(\alpha - 0) = \pi^{-1}\int\limits_0^\infty \cos\lambda\alpha \, d\lambda$ is the Dirac function.

We expand the even $2\pi/k$-periodic function in a cosine series:

$$q(\beta) = \sum_{n=0}^{\infty} q_n \cos kn\beta. \tag{3}$$

The coefficients are written as the integrals

$$q_0 = \frac{k}{\pi}\int\limits_0^{\beta_0} f(\beta)\, d\beta = \frac{kP}{2\pi R}, \quad q_n = \frac{2k}{\pi}\int\limits_0^{\beta_0} f(\beta)\cos kn\beta \, d\beta. \tag{4}$$

For each of the harmonics n, we construct solutions of Eq. (1) that decrease at $\pm\infty$. Such a solution does not exist for $n = 0$. For this reason, the axisymmetric component is taken into account only in the boundary effect, while for the principal state we assume

$$\frac{\partial^4\Phi}{\partial\alpha^4} + c^2\frac{\partial^8\Phi}{\partial\beta^8} = \frac{R}{\pi Eh}\sum_{n=1}^{\infty} q_n \cos kn\beta \int\limits_0^\infty \cos\lambda\alpha \, d\lambda.$$

This equation has the solution

$$\Phi(\alpha,\beta) = \frac{R}{\pi Eh}\sum_{n=1}^{\infty} q_n \cos kn\beta \int\limits_0^\infty \frac{\cos\lambda\alpha}{\lambda^4 + c^2(kn)^8}\, d\lambda,$$

from which, for the force factors, in accordance with (2), we have

$$\begin{aligned} G_1^{\mathrm{p}} &= \nu G_2^{\mathrm{p}} = \frac{\nu D}{\pi REh}\sum_{n=1}^{\infty} q_n (kn)^6 \cos kn\beta \int\limits_0^\infty \frac{\cos\lambda\alpha}{\lambda^4 + c^2(kn)^8}\, d\lambda, \\ T_1^{\mathrm{p}} &= -\frac{1}{\pi}\sum_{n=1}^{\infty} q_n (kn)^2 \cos kn\beta \int\limits_0^\infty \frac{\lambda^2\cos\lambda\alpha}{\lambda^4 + c^2(kn)^8}\, d\lambda. \end{aligned} \tag{5}$$

We restrict ourselves only to the force factors in the loaded cross section $\alpha = 0$, in which they reach maximum values. Taking into account that [7]

$$\int\limits_0^\infty \frac{\lambda^2\, d\lambda}{\lambda^4 + c^2(kn)^8} = \frac{\pi\sqrt{2}}{4(kn)^2\sqrt{c}}, \quad \int\limits_0^\infty \frac{d\lambda}{\lambda^4 + c^2(kn)^8} = \frac{\pi\sqrt{2}}{4(kn)^6 c\sqrt{c}}, \tag{6}$$

we obtain simpler expressions in place of (5):

$$T_1^{\mathrm{p}} = -\frac{1}{2\sqrt{2c}}(q(\beta) - q_0), \quad G_1^{\mathrm{p}} = \nu G_2^{\mathrm{p}} = \frac{\nu R\sqrt{c}}{2\sqrt{2}}(q(\beta) - q_0). \tag{7}$$

These relations are written with allowance for series (3).

Solutions of the boundary-effect equation that decrease at $\pm\infty$ are well known [8] and will not be written here in complete form. In the loaded cross section ($\alpha = 0$), they have the form

$$T_1^{\mathrm{b}} = 0, \quad T_2^{\mathrm{b}} = -\frac{1}{2}q(\beta)\sqrt[4]{3(1-\nu^2)}\sqrt{R/h}, \quad G_2^{\mathrm{b}} = \nu G_1^{\mathrm{b}} = \frac{\nu}{4}q(\beta)\sqrt{Rh}/\sqrt[4]{3(1-\nu^2)}. \tag{8}$$

Accomplishing the synthesis of stress states (7) and (8), we write relations for forces and moments:

$$
\begin{aligned}
T_1(0,\beta) &= T_1^{\mathrm{p}} + T_1^{\mathrm{b}} = -\frac{1}{2}\sqrt[4]{3(1-\nu^2)}\sqrt{\frac{R}{h}}\Big(q(\beta) - \frac{k}{\pi}\int_0^{\beta_0} f(\beta)\,d\beta\Big),\\
T_2(0,\beta) &= T_2^{\mathrm{p}} + T_2^{\mathrm{b}} = -\frac{1}{2}\sqrt[4]{3(1-\nu^2)}\sqrt{\frac{R}{h}}q(\beta),\\
G_1(0,\beta) &= G_1^{\mathrm{p}} + G_1^{\mathrm{b}} = \frac{\sqrt{Rh}}{4\sqrt[4]{3(1-\nu^2)}}\Big((1+\nu)q(\beta) - \frac{\nu k}{\pi}\int_0^{\beta_0} q(\beta)\,d\beta\Big),\\
G_2(0,\beta) &= G_2^{\mathrm{p}} + G_2^{\mathrm{b}} = \frac{\sqrt{Rh}}{4\sqrt[4]{3(1-\nu^2)}}\Big((1+\nu)q(\beta) - \frac{k}{\pi}\int_0^{\beta_0} q(\beta)\,d\beta\Big).
\end{aligned}
\tag{9}
$$

It follows from these relations that the distributions of the force factors in the stress state with respect to the angular coordinate differ from the external-load distribution only by the multiplicative term and constant. The latter distribution can be quite arbitrary but admits a converging cosine Fourier series expansion.

As an example, we consider an external load with density

$$
f(\beta) = \frac{P\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)\beta_0 R}\Big(1 - \frac{\beta^2}{\beta_0^2}\Big)^{\mu}, \quad \beta \in [-\beta_0, \beta_0], \tag{10}
$$

where $\Gamma(z)$ is the Euler function; z is an argument, e.g., $z = \mu + 1$; and $\mu \geqslant 0$. The restriction on μ is due to the Dirichlet theorem on a Fourier series expansion of a function. Relation (10), in particular, yields a uniform distribution ($\mu = 0$), a Hertz-type distribution ($\mu = 1/2$), etc.

We compute the integral entering into (9):

$$
\int_0^{\beta_0} f(\beta)\,d\beta = \frac{P\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)\beta_0 R}\int_0^{\beta_0}\Big(1 - \frac{\beta^2}{\beta_0^2}\Big)^{\mu} d\beta = \frac{P\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)\beta_0 R}\int_0^{\pi/2} \cos^{2\mu+1} t\,dt = \frac{P}{2R}.
$$

Here we take into account that [7]

$$
\int_0^{\pi/2} \cos^{2\mu+1} t\,dt = 2^{2\mu}\frac{(\Gamma(\mu+1))^2}{\Gamma(2\mu+2)}, \quad \Gamma(2\mu+2) = \frac{2^{2\mu+1}}{\sqrt{\pi}}\Gamma(\mu+1)\Gamma(\mu+3/2).
$$

As one can see, the chosen power distribution of load (10) satisfies condition (4). In this case, using relations (9) for the force factors, for the center of the loaded segment we obtain

$$
\begin{aligned}
T_1(0,0) &= -\frac{P\sqrt[4]{3(1-\nu^2)}}{2\beta_0 R}\sqrt{\frac{R}{h}}\Big(\frac{\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)} - \frac{k\beta_0}{2\pi}\Big),\\
T_2(0,0) &= -\frac{P\sqrt[4]{3(1-\nu^2)}}{2\beta_0 R}\sqrt{\frac{R}{h}}\frac{\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)},\\
G_1(0,0) &= \frac{P}{4\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{h}{R}}\Big[(1+\nu)\frac{\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)\beta_0} - \frac{\nu k}{2\pi}\Big],\\
G_2(0,0) &= \frac{P}{4\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{h}{R}}\Big[(1+\nu)\frac{\Gamma(\mu+3/2)}{\sqrt{\pi}\Gamma(\mu+1)\beta_0} - \frac{k}{2\pi}\Big].
\end{aligned}
\tag{11}
$$

These expressions can be further simplified for constant load density. When $\mu = 0$, $\Gamma(3/2) = \sqrt{\pi}/2$,

TABLE 1

ω	γ	t_1	t_2	g_1	g_2
1.1	0.5	1.40	1.41	1.41	1.40
2.2	1.0	0.69	0.71	0.70	0.69
3.3	1.5	0.45	0.43	0.47	0.46
4.4	2.0	0.33	0.35	0.34	0.37

TABLE 2

ω	γ	t_1	t_2	g_1	g_2
1.1	0.5	0.69	0.69	1.35	1.35
2.2	1.0	0.55	0.55	0.77	0.77
3.3	1.5	0.43	0.43	0.51	0.51
4.4	2.0	0.34	0.34	0.37	0.37

and $\Gamma(1) = 1$, we have

$$T_1(0,0) = -\frac{P\sqrt[4]{3(1-\nu^2)}}{4\pi\beta_0\sqrt{Rh}}(\pi - k\beta_0), \quad T_2(0,0) = -\frac{P\sqrt[4]{3(1-\nu^2)}}{4\beta_0\sqrt{Rh}},$$

$$G_1(0,0) = \frac{P}{8\pi\beta_0\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{h}{R}}[(1+\nu)\pi - \nu k\beta_0], \tag{12}$$

$$G_2(0,0) = \frac{P}{8\pi\beta_0\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{h}{R}}[(1+\nu)\pi - k\beta_0].$$

Relations (12) are completely identical to those in [1, 2]. Thus, solutions (9) and (11) should be regarded as extensions of the known results (12) to the case of a nonuniform distribution of the load along the segments of the directing circle.

In constructing approximate solutions herein, the synthesis of the stress state was performed only for two components, that is, the third (flexural) component was disregarded. It is demonstrated in [1, 2] that this is allowable in calculation of G_1 and G_2 for a load distributed uniformly along segments, provided that $\beta_0 \geqslant \sqrt{h/R}$. In this connection, the question arises of whether it is possible to extend this inequality to the case of a nonuniform distribution. To get an answer, we compare values of the forces and moments obtained from (11) for $\mu = 1/2$ with those obtained in [5] using the two-dimensional Fourier transform based on the Vlasov–Donnel equations. Solving the problem for dimensionless forces and moments at $\mu = 1/2$, $k = 1$, $\alpha = 0$, and $\beta = 0$, we come to the expressions

$$t_j = -\frac{2\pi h T_j(0,0)}{P\sqrt{3(1-\nu^2)}} = -(B_0V_0 + C_0U_0 + B_1V_1 + C_1U_1),$$
$$g_j = \frac{4\pi G_j(0,0)}{P(1+\nu)} = B_0U_0 - C_0V_0 + B_1U_1 - C_1V_1 \quad (j = \overline{1,2}), \tag{13}$$

where $B_m = \mathrm{ber}_m(\gamma)$; $C_m = \mathrm{bei}_m(\gamma)$; $U_m = \mathrm{ker}_m(\gamma)$; $V_m = \mathrm{kei}_m(\gamma)$; $\gamma = (1/2\sqrt{2})\beta_0\sqrt[4]{3(1-\nu^2)}\sqrt{R/h}$; B_m, C_m, U_m, and V_m are Thompson functions.

For comparative analysis, we assume that $\nu = 0.3$, $R/h = 400$, and $\beta_0 = \omega\sqrt{h/R}$. The results of calculations by formulas (11) and (13) are presented in Tables 1 and 2, respectively. For values of the Thompson functions we refer to [9]. Analysis shows that for a nonuniform load distribution (in particular, by the Hertz law) the validity of the inequality $\beta_0 \geqslant \sqrt{h/R}$ (or $\omega \geqslant 1$) suggested in [1] gives satisfactory agreement between the moments obtained using the ASM and those obtained by means of equations of the technical moment theory of shells. For tangential forces, such agreement is achieved for loaded segments of much greater lengths, i.e., starting with $\beta_0 \geqslant 3\sqrt{h/R}$ (or $\omega \geqslant 3$). Within the framework of the above inequalities, the advantages of the ASM are obvious, since it yields simpler closed solutions which are sufficiently exact and convenient for engineering calculations.

We now consider an infinitely long shell under the action of a normal load which is piecewise constant in the longitudinal direction and piecewise linear in the circumferential direction. For such a load, Fourier series expansions along the contour and representation by a Fourier integral in the longitudinal directions are valid:

$$p(\alpha,\beta)=\frac{4}{\pi^2 k\beta_0}p_0\sum_{n=1}^{\infty}\omega_n\sin kn\beta\int_0^{\infty}\frac{1}{\lambda}\sin\alpha_0\lambda\cos\alpha\lambda\,d\lambda. \tag{14}$$

Here p_0 is the amplitude of the normal load; α_0 is a parameter that characterizes the extent of the loaded region along the ruling (the length of the region is $2\alpha_0 R$), and $\omega_n=(1/n^2)(\sin kn\beta_0-kn\beta_0\cos kn\beta_0)$.

Using the approximate equations describing elementary stress states (the principal state, the boundary effect in the zone to which the load is applied, and the bending state), we obtain a solution of the problem for the particular case of a load applied along the contour segments where $\alpha_0\to 0$. The total stress-strain state of the shell is represented as a sum of three elementary states: the principal state, the local boundary effect (when $n\leqslant n^*$), and the bending state (when $n>n^*$). This constitutes, in essence, the third ASM [2]. Thus, we find the following expressions for the most important factors of the stress-strain state of the shell:

$$\frac{ER^2}{M_0}w(\alpha,\beta)=\frac{9(1-\nu^2)}{\pi\sqrt[4]{3(1-\nu^2)}\beta_0^3}\left(\frac{R}{h}\right)^2\sqrt{\frac{R}{h}}\sum_{n=1}^{n^*}\frac{n\omega_n}{\sqrt{(k^2n^2-1)^3}}[\varphi_n(\alpha)+\psi_n(\alpha)]\sin kn\beta$$

$$+\frac{3\sqrt[4]{3(1-\nu^2)}}{4\beta_0^2}\frac{R}{h}\sqrt{\frac{R}{h}}[\varphi(\alpha)+\psi(\alpha)]\omega^*(\beta)+\frac{9(1-\nu^2)}{\pi k^4\beta_0^3}\left(\frac{R}{h}\right)^3\sum_{n^*+1}^{\infty}\frac{\omega_n}{n^3}(1+kn\alpha)\mathrm{e}^{-kn\alpha}\sin kn\beta,$$

$$\frac{R^2}{M_0}T_1(\alpha,\beta)=-\frac{3\sqrt[4]{3(1-\nu^2)}}{2\pi\beta_0^3}\sqrt{\frac{R}{h}}\sum_{n=1}^{n^*}\frac{n\omega_n}{\sqrt{k^2n^2-1}}[\varphi_n(\alpha)-\psi_n(\alpha)]\sin kn\beta,$$

$$\frac{R^2}{M_0}T_2(\alpha,\beta)=-\frac{3\sqrt[4]{3(1-\nu^2)}}{4\beta_0^2}\sqrt{\frac{R}{h}}[\varphi(\alpha)+\psi(\alpha)]\omega^*(\beta), \tag{15}$$

$$\frac{R}{M_0}G_1(\alpha,\beta)=\frac{3}{8\sqrt[4]{3(1-\nu^2)}\beta_0^2}\sqrt{\frac{h}{R}}[\varphi(\alpha)-\psi(\alpha)]\omega^*(\beta)$$

$$+\frac{3\nu}{4\pi\sqrt[4]{3(1-\nu^2)}\beta_0^3}\sqrt{\frac{h}{R}}\sum_{n=1}^{n^*}\frac{n\omega_n}{\sqrt{k^2n^2-1}}[\varphi_n(\alpha)+\psi_n(\alpha)]\sin kn\beta$$

$$+\frac{3}{4\pi k^2\beta_0^3}\sum_{n^*+1}^{\infty}\frac{\omega_n}{n}[1+\nu-(1-\nu)kn\alpha]\mathrm{e}^{-kn\alpha}\sin kn\beta,$$

$$\frac{R}{M_0}G_2(\alpha,\beta)=\frac{3}{4\pi\sqrt[4]{3(1-\nu^2)}\beta_0^3}\sqrt{\frac{h}{R}}\sum_{n=1}^{n^*}\frac{n\omega_n}{\sqrt{k^2n^2-1}}[\varphi_n(\alpha)+\psi_n(\alpha)]\sin kn\beta$$

$$+\frac{3\nu}{8\sqrt[4]{3(1-\nu^2)}\beta_0^2}\sqrt{\frac{h}{R}}[\varphi(\alpha)-\psi(\alpha)]\omega^*(\beta)+\frac{3}{4\pi k^2\beta_0^3}\sum_{n^*+1}^{\infty}\frac{\omega_n}{n}[1+\nu+(1-\nu)kn\alpha]\mathrm{e}^{-kn\alpha}\sin kn\beta,$$

$$\omega^*(\beta)=\frac{2}{\pi k\beta_0}\sum_{n=1}^{n^*}\omega_n\sin kn\beta.$$

Here M_0 is the total moment transmitted to the shell through one of the k contour segments (the moment is equivalent to the normal load); the functions $\varphi_n(\alpha)$, $\psi_n(\alpha)$, $\varphi(\alpha)$, and $\psi(\alpha)$ are determined from the formulas [2] $\varphi_n(\alpha)=\exp(-\mu_n\alpha)\cos\mu_n\alpha$, $\psi_n(\alpha)=\exp(-\mu_n\alpha)\sin\mu_n\alpha$, $\varphi(\alpha)=\exp(-\eta\alpha)\cos\eta\alpha$, and $\psi(\alpha)=\exp(-\eta\alpha)\sin\eta\alpha$.

Note that the harmonic number n^* for "gluing" of the solutions of the principal state, boundary effect, and bending state written in the form of (15) is found by using the well-known formula relating the harmonic

TABLE 3

n	General theory of shells	Principal state	Local boundary effect	Principal state plus boundary effect	Bending state
			G_{2n}/P		
1	0.0170	0.0137	0.0036	0.0173	0.0986
2	0.0144	0.0108	0.0031	0.0139	0.0430
3	0.0118	0.0084	0.0025	0.0109	0.0225
4	0.0080	0.0058	0.0017	0.0075	0.0115
5	0.0040	0.0030	0.0009	0.0039	0.0048
10	−0.0018	−0.0024	−0.0007	−0.0031	−0.0019
15	0.0008	0.0015	0.0005	0.0020	0.0008
20	−0.0003	−0.0006	−0.0002	−0.0008	−0.0003
30	0.0001	0.0002	0.0002	0.0004	0.0001
40	0.0001	0.0001	0.0002	0.0003	0.0001

number, the number of loads, and the relative thickness of the shell [2]:

$$n^4 \approx (2/k^4)(1-\nu^2)(R/h)^{5/2}. \tag{16}$$

The value of n obtained from (16) and rounded to the nearest integer is the desired n^*. The values of n^* calculated from the above formula are identical to the corresponding values determined by a numerical experiment for various parameters k, h/R, and β_0. To illustrate this, we give numerical data for a shell with relative thickness $h/R = 1/100$ loaded by two diametrically opposite loads for which $k = 2$ and $\beta_0 = 0.25$. The maximum values for the annular bending moment G_{2n}/P are presented in Table 3 for various n (from $n = 1$ to $n = 40$). Some intermediate values are omitted to save space, but this does not make the table less informative. The numerical information was obtained from the equations of the general theory of shells in [10], the equations of semi-momentless theory (the principal state), and the equations of the boundary effect and bending state.

As for a piecewise constant load along the contour [1, 2] and for a sufficiently arbitrary load symmetrical about the coordinate origin (convenient formulas have been obtained herein for the latter case), it is of interest to construct simple calculation formulas for the case at hand, i.e., when the load is statically equivalent to one or more local circumferential moments. Simple computation formulas suitable for a certain range of the load parameters k and β_0 can be derived by simplification of the sufficiently general solution (15). In this case, as before, we assume that in a certain range of the parameter β_0 the solution can be written solely on the basis of semi-momentless and boundary-effect equations. Solutions (15) for the principal state and the local-boundary effect written as series are then assumed to be valid over the entire range of harmonic numbers: $n = 1, 2, 3, \ldots, n^*, n^*+1, \ldots, \infty$. This becomes possible owing to the fact that in a certain range of the parameter β_0 the partial sum (starting with a certain $n > n^*$ up to infinity) makes no substantial contribution to the total sum of the series. Since the series become infinite, it becomes possible, in some particular cases, to find their sum in finite form and to write simple formulas without series for the desired force and strain factors. Naturally, in this case, the region of applicability of the resulting formulas is narrower in comparison with solution (15), and it becomes necessary to establish the boundaries of the region, for example, by comparison with the exact data, as is done later. It is seen from Table 3 that for large harmonic numbers, the error in determining the bending moment using semi-momentless theory can be substantial if the shell is acted on by a sinusoidal (or cosinusoidal) load rather than a localized load considered herein. For instance, when the load is $p_0 \sin 30\beta$, the solution yielded by semi-momentless theory differs by a factor of two from the exact solution (yielded by the general theory of shells), while the bending state yields a value that is completely identical to that obtained in general theory.

In the case of an infinitely long shell, the equilibrium condition implies that $k \geqslant 2$. If we assume a strong inequality $k^2n^2 \gg 1$, which means transition from the resolving equation of semi-momentless theory

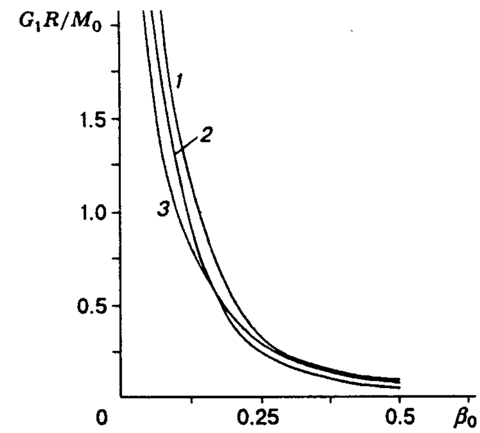

Fig. 1

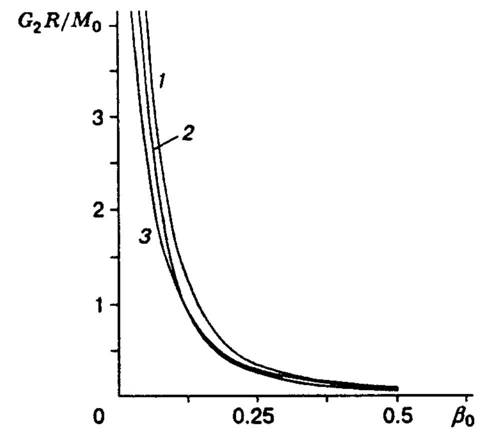

Fig. 2

to the simplified equation (1), for $\alpha = 0$, we have

$$\frac{R^2}{M_0}T_1^{\mathrm{p}}(0,\beta) = -\frac{3\sqrt[4]{3(1-\nu^2)}}{2\pi k\beta_0^3}\sqrt{\frac{R}{h}}\sum_{n=1}^{\infty}\left(\frac{\sin kn\beta_0}{n^2} - k\beta_0\frac{\cos kn\beta_0}{n}\right)\sin kn\beta,$$

$$\frac{R}{M_0}G_2^{\mathrm{p}}(0,\beta) = \frac{3}{4\pi k\sqrt[4]{3(1-\nu^2)}\beta_0^3}\sqrt{\frac{h}{R}}\sum_{n=1}^{\infty}\left(\frac{\sin kn\beta_0}{n^2} - k\beta_0\frac{\cos kn\beta_0}{n}\right)\sin kn\beta.$$

The series that enters into the expressions for the longitudinal forces and bending moments can be summed up. Then, after certain transformations for $\beta = \beta_0$, i.e., for the boundary of the loaded region, we obtain the final formulas

$$\frac{R^2}{M_0}T_1^{\mathrm{p}}(0,\beta_0) = -\frac{3\sqrt[4]{3(1-\nu^2)}}{8\beta_0^2}\sqrt{\frac{R}{h}}, \quad \frac{R}{M_0}G_2^{\mathrm{p}}(0,\beta_0) = \frac{3}{16\sqrt[4]{3(1-\nu^2)}\beta_0^2}\sqrt{\frac{h}{R}}. \tag{17}$$

Similar formulas can be found for the local boundary effect. For $\beta = \beta_0$, they take the form

$$\frac{R^2}{M_0}T_2^{\mathrm{b}}(0,\beta_0) = -\frac{3\sqrt[4]{3(1-\nu^2)}}{8\beta_0^2}\sqrt{\frac{R}{h}}, \quad \frac{R}{M_0}G_1^{\mathrm{b}}(0,\beta_0) = \frac{3}{16\sqrt[4]{3(1-\nu^2)}\beta_0^2}\sqrt{\frac{h}{R}}. \tag{18}$$

Thus, the longitudinal force in the shell is determined as a solution of the principal state $T_1(\alpha,\beta) \approx T_1^{\mathrm{p}}(\alpha,\beta)$, and the annular force as a solution for the local boundary effect $T_2(\alpha,\beta) \approx T_2^{\mathrm{b}}(\alpha,\beta)$. Complete formulas for the bending moments are obtained by summation of the solutions for the local boundary effect (18) and for the principal state (17). For the moments at the boundary $\beta = \beta_0$ of the loaded region, we find

$$\frac{R}{M_0}G_1(0,\beta_0) = \frac{R}{M_0}G_2(0,\beta_0) = \frac{3(1+\nu)}{16\sqrt[4]{3(1-\nu^2)}\beta_0^2}\sqrt{\frac{h}{R}}. \tag{19}$$

It is of interest to compare the numerical results yielded by the approximate formulas and the exact solution to refine the region of their applicability. Good agreement is obtained between the data yielded by formula (19) and the exact solution over a wide range of the loaded-region parameter $\beta_0 \geqslant (h/R)^{1/2}$. For smaller values of the parameter, the difference can become more substantial. It then becomes necessary to take into account the bending state, which is a component of the solution (15) and becomes more and more powerful as β_0 decreases. As $\beta_0 \to 0$, this state begins to play a determining role. This can be easily seen from the dependences of the longitudinal (Fig. 1) and annular (Fig. 2) bending moments on the parameter β_0 of the extent of the loaded region for $k = 2$ and $R/h = 100$. The solution given by formula (15) is shown by curve 1, and that provided by (19) is shown by curve 2. Curve 3 corresponds only to the solutions of equations of the boundary effect (Fig. 1) or equations of the semi-momentless theory of shells (Fig. 2). These solutions are written as formulas (18) and (17), respectively. Thus, curve 3 characterizes the contribution of the boundary effect and the principal state to the total stress state (to the total value of the bending moments which are the main force factors). Note that solution (15) shown by curve 1 in Figs. 1 and 2 is completely identical to the solution provided by the general theory of shells (the error does not exceed 5%).

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